STATISTICAL MODEL FOR SCATTERING MATRICES OF OPEN CAVITIES

Thomas M. Antonsen, Xing Zheng, Edward Ott
Institute for Research in Electronics and Applied Physics,
Sameer Hammedy, Steven Anlage
Center for Superconductivity Research,
University of Maryland College, Park MD, 20742, USA

Abstract: We propose a model to study the statistical properties of the impedance (Z) and scattering (S) matrices of open electromagnetic cavities with several transmission lines or waveguides connected to the cavity. The model is based on assumed properties of chaotic eigenfunctions for the closed system. Statistical properties of the cavity impedance Z are obtained in terms of the radiation impedance (i.e., the impedance seen at a port with the cavity walls moved to infinity). Effects of wall absorption and nonreciprocal media (e.g., magnetized ferrite) are discussed. Theoretical predictions are tested by direct comparison with numerical solutions for a specific system.

INTRODUCTION

The problem of the coupling of electromagnetic radiation in and out of structures is a general one that finds applications in a variety of scientific and engineering contexts. Examples include the susceptibility of circuits to electromagnetic interference, the confinement of radiation to enclosures, as well as the coupling of radiation to accelerating structures. Because of the wave nature of radiation, the coupling properties of a structure depend in detail on the size and shape of the structure, as well as the frequency of the radiation. In considerations of irregularly shaped electromagnetic enclosures for which the wavelength is fairly small compared with the size of the enclosure, it is typical that the electromagnetic field pattern within the enclosure, as well as the response to external inputs, can be very sensitive to small changes in frequency and to small changes in the configuration. Thus, knowledge of the response of one configuration of the enclosure may not be useful in predicting that of a nearly identical enclosure. This motivates a statistical approach to the electromagnetic problem [1].

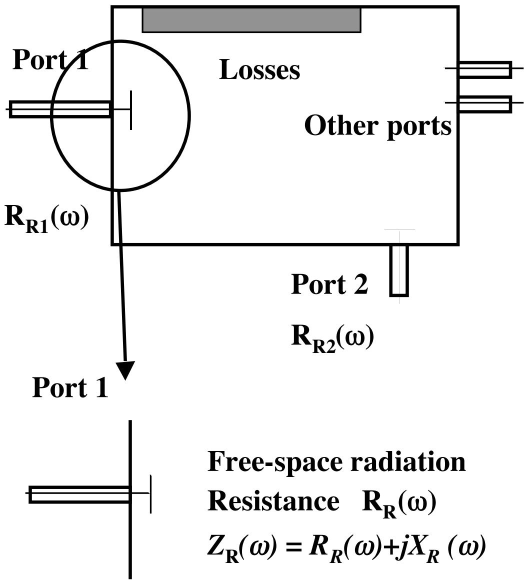

Fig. 1 Schematic illustrating application of the random coupling model

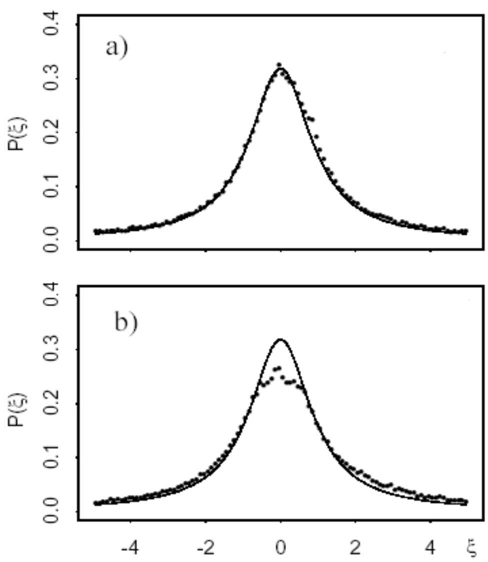

Fig.2 Histograms of normalized cavity reactance for a) 6.75-7.25 GHz and b) 7.75 to 8.25 GHz

We have developed a statistical approach [2], which we call the random coupling model, to describe the properties of a high-frequency microwave cavity with several ports and losses. We express the scattering matrix for this system using the cavity impedance matrix. The impedance matrix is derived in terms of the eigenfunctions and eigenfrequencies of the closed

cavity. Explicit calculation of the eigenfunctions and eigenfrequencies is not required however. Rather, in view of the extreme sensitivity of these to the specific geometry they are replaced by functions drawn from a statistical ensemble. We find that the impedance matrix can then be expressed in terms of random variables with well-defined statistics and relatively simple, physical quantities characterizing the cavity.
The method of application of our model is illustrated in Fig. 1. One first isolates the ports of interest, in this case port 1 and port 2, and computes (or measures) the free space radiation impedance for each port. The process of isolation consists of determining what is in the near field region of the port and including it in the calculation of the radiation impedance. The concept of a port can be generalized to apply to terminals on circuits within the enclosure. Each port is then characterized by the free space radiation resistance $R_{Ri}(\omega)$. The additional important physical quantities needed in our model are the volume of the cavity and the cavity quality factor. The impedance matrix is then modeled by the formula,

$$Z_{ij}(\omega) = -\frac{j}{\pi}\sum_n R_{Ri}^{1/2}(\omega_n) R_{Rj}^{1/2}(\omega_n) \frac{\Delta\omega_n^2 w_{in} w_{jn}}{\omega^2(1-j/Q)-\omega_n^2},$$

where, $\Delta\omega_n^2$ is the mean spectral density of the cavity, Q is the average quality factor, w_{in} are a set of independent, zero mean, unit variance Gaussian random numbers, and ω_n^2 is a random spectrum determined by generating random spacings between eigenfrequencies consistent with the average spectral density. We describe in the next sections some of the predictions of this model.

CAVITY IMPEDANCE DISTRIBUTIONS

We have parameterized the probability distribution function for the real and imaginary parts of the cavity impedance. For a lossless, single port cavity the impedance is imaginary with a mean and fluctuating part. The mean part is equal to the radiation reactance for the port under the conditions of radiation into free space. The fluctuating part of the reactance is Lorenzian distributed, with a width given by the free space radiation resistance for the port. To test this prediction we have solved the for the field distribution inside an irregularly shaped cavity with a moveable obstacle, driven by a coaxial transmission line using HFSS.

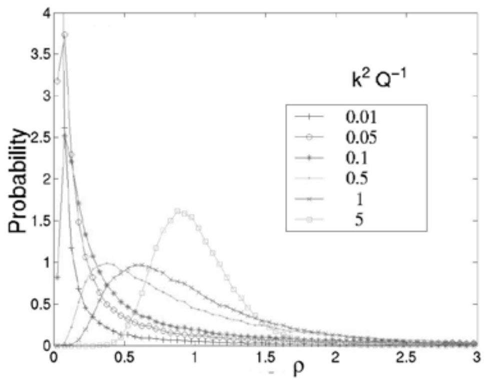

Fig. 3 Histogram of normalized resistance $\rho=R(\omega)/R_R(\omega)$

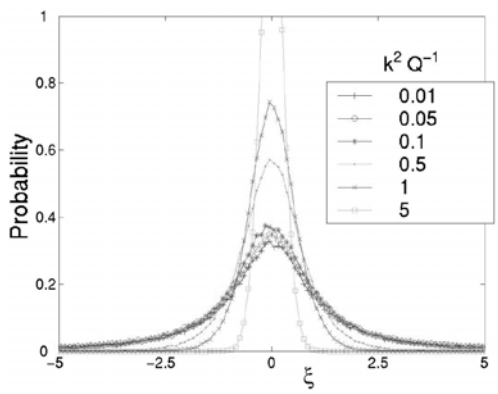

Fig.4 Histograms of normalized cavity reactance ξ

Calculations were made for 100 positions of the obstacle and 4000 frequencies. Histograms of the normalized reactance defined as $\xi = (X(\omega)-X_R(\omega)/R_R(\omega)$ are plotted in two frequency ranges in Fig. 2 along with the predicted unit Lorenzian. The histograms are seen to approach the predicted shape, however, there is an anomaly in curve b) that we attribute to the effect of strong reflections in our cavity which are not eliminated by the moveable obstacle. This anomaly disappears if a large enough frequency range is considered.

When losses are added, or when additional ports are added which couple energy out of the cavity, the cavity impedance becomes complex. The distribution of values can then be parameterized in terms of the free space radiation impedance of the port and the cavity quality factor. In terms of the statistics of cavity impedance values, there is an equivalence between the cases of distributed losses and localized losses at output ports. Predicted histogram plots of the imaginary and real parts of the cavity impedance (normalized to the real part of the radiation impedance) are shown in Figs. 3 and 4. The different curves correspond to different values of cavity quality factor and show the transition from the case of a lossless cavity to that of essentially radiation into free space.

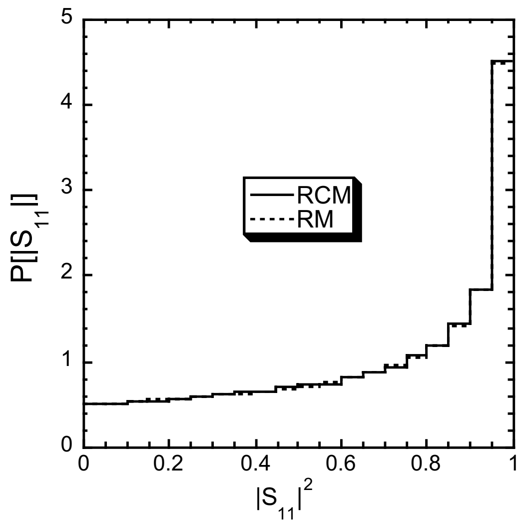

Fig. 5 Histogram of reflection coefficient for the random coupling model (RCM) and the random matrix model (RM) in the case of a reciprocal medium

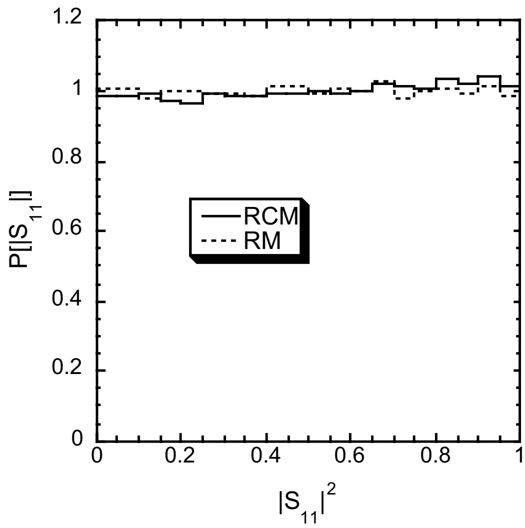

Fig. 6 Histogram of reflection coefficient for the random coupling model (RCM) and the random matrix model (RM) in the case of a nonreciprocal medium

TWO-PORT TRANSMISSION COEFFICIENTS

The statistics of the scattering matrix for lossless, complex systems is frequently characterized in terms of a random matrix. This approach is used in nuclear and condensed matter physics as well as in wave chaos theory [3]. An important requirement of our random coupling model is that it give identical results to the random matrix approach when applicable. We have tested this by generating scattering random matrices using both the random coupling model (RCM) and the random matrix model (RM). We have considered two cases of interest. One in which the underlying wave propagation is reciprocal and one in which a nonreciprocal element such as a ferrite is included. Figures 5 and 6 show histograms of the reflection coefficient generated by the two approaches in the two cases. As can be seen the random coupling model and the random matrix model produce the same results.

CONCLUDING REMARKS

Finally, we note that comparisons with experimentally measured scattering coefficients are underway and we hope to report these soon. This work was supported by the MURI "The Effects of RF Pulses on Electronic Circuits and Systems", Administered by the US AFOSR.

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